REFLECTIONS ON BRAGG'S LAW

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On January 9, Andrew Lloyd Webber's *Phantom of the Opera* overtook that composer's own record-setting *Cats* to become the longest-playing musical on Broadway. After opening in 1988, *Phantom* has glided through more than 7,500 performances; that's eight times a week over eighteen years. The worldwide box-office receipts for this show exceed $3.2 billion, which, according to the *Washington Post*, makes it the highest-grossing artistic enterprise in history (nearly double the $1.8 billion reaped by the blockbuster movie *Titanic*). The statistics needed to capture the magnitude of this achievement are akin to those we cite for geologic time scales. The playbills from *Phantom*, for instance, laid end to end would extend over 43,000 miles.

The human measure of this marathon is, of course, the most amazing part of the story. Three actors have played the show continuously since opening night, and many more can boast thousands of stage appearances. For a profession in which 90% of its union members are unemployed at any given moment, *Phantom* has provided the closest thing to tenure that most performance artists are likely to know.

The question that pops into one's mind is how these actors manage to keep it fresh night after never-ending night. Ted Williams, the slugger for the Boston Red Sox and the last to end a baseball season with higher than a .400 average, accounted for his intensity this way: he imagined at the beginning of every game that someone in the stands had never seen him bat before, and he intended to show that person what he could do at the plate. One supposes that the actors in *Phantom* have succeeded in the musical equivalent of this self-hypnosis.

The numbing crush of repetitive performance is hardly unknown to most readers of this journal. Perhaps the thought of singing "Music of the Night" for the four thousandth time may go some way toward quelling our exasperation as we prepare to teach the Bravais lattices to yet another class of jaded undergraduates. But who among us has never despaired at the prospect of resounding the worn strains of a hoary lecture for an audience that increasingly demands Broadway-scale theatrics as the delivery vehicle for its consumption of Pauling's Rules?

It was in this frame of mind some years ago that I calculated the number of times I would have to derive Bragg's Law over the remainder of my career, and somehow I came up with the number 43. Even at that pre-*Phantom*-record-breaking moment, I reasoned that 43 iterations of this foundational diffraction relationship paled in comparison with, say, the number of times that a member of a string quartet might have to perform *Eine Kleine Nachtmusik* at weddings and bar mitzvahs over the course of a decade. But we are not violists. We are people who are supposed to embody Tennyson's injunction: "To follow knowledge like a sinking star/Beyond the utmost bound of human thought." Regurgitating Bragg's Law 43 more times seemed the opposite of that.

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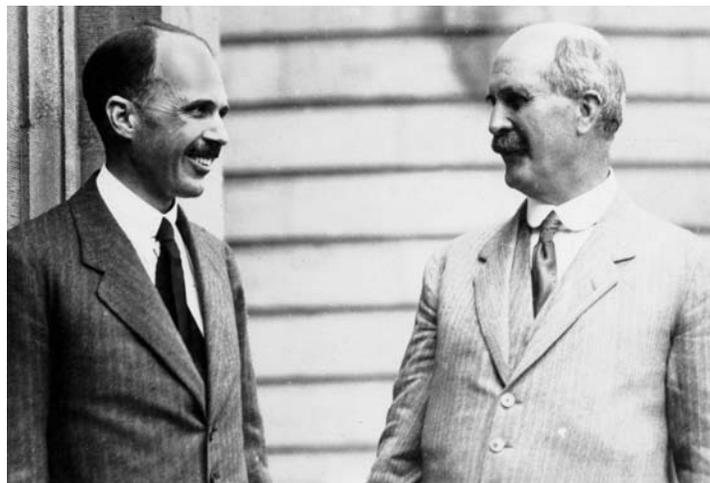
FIGURE 1 A snapshot of William Lawrence Bragg (left) and William Henry Bragg (right). COURTESY OF THE EDGAR FAHS SMITH COLLECTION, UNIVERSITY OF PENNSYLVANIA LIBRARY

Science histories offer the best candles to light one's way out of this kind of darkness, and a new biography of William Lawrence Bragg by Graeme K. Hunter paints a life of intellectual excitement that vastly exceeds anything conceived by Andrew Lloyd Webber (with the probable exception of *Jesus Christ, Superstar*). Entitled *Light is a Messenger*, this volume provides the first full overview of the youngest person ever to win a Nobel Prize. Bragg was 25 when he was named the 1915 Nobel laureate in physics. It is nothing short of astounding that nearly a century passed between the time Bragg formulated his famous relationship and the year a biography of the man was published. It is true that 1990 marked the appearance of *Selections and Reflections: The Legacy of Sir Lawrence Bragg*, but this book is only what its title promises—an undigested compendium of remembrances of Bragg by those who knew him. It really is the primary material from which a biography would have to be woven.

One hopes that *Messenger* is the first of several portraits, since it strongly caters to those who employ structure factors as part of their workaday vocabulary. But even in this rendering, the human side of Bragg manages to break through the frosty reserve that confronts any investigator of his life. He was susceptible to episodes of depression and moments of explosive anger. His relationship with his father was psychologically tortuous. William Henry Bragg himself was a brilliant experimentalist who laid much of the groundwork for X-ray spectroscopy, and the collaboration between W.H. and W.L. was a complex amalgam of professional rivalry and loving esteem (FIG. 1).

In fact, the discovery of the famous law turns out to have originated in an effort by the son to save a flawed theory of the father from an unexpected attack. In 1907, W.H. hypothesized that the ability of X-rays to eject electrons from atoms during the ionization of gases could be explained only if X-rays were corpuscular—specifically, if X-rays represented neutral pairs of α and β particles. A serious challenge to this model appeared in 1912, when Max von Laue reported that his assistants Walter Friedrich and Paul Knipping had successfully diffracted X-rays from crystals of copper sulfate and, more convincingly, from sphalerite.

In the early fall of 1912, Bragg father and son set out to demonstrate that Laue's experiment actually confirmed the neutral-pair theory. They reasoned that the intense spots that had appeared on Laue's photographic plates resulted from a channeling of X-ray particles along open avenues outlined by rows of atoms in an oriented sphalerite crystal. They reproduced Laue's observations in W.H.'s laboratory in Leeds, but then they tilted the crystal, with the expectation that no spots would appear on the plates when the open tunnels were oblique to the narrow X-ray beam. The test provided no evidence for the channeling of particles.

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$$\lambda = 2\,d\,\sin\theta$$
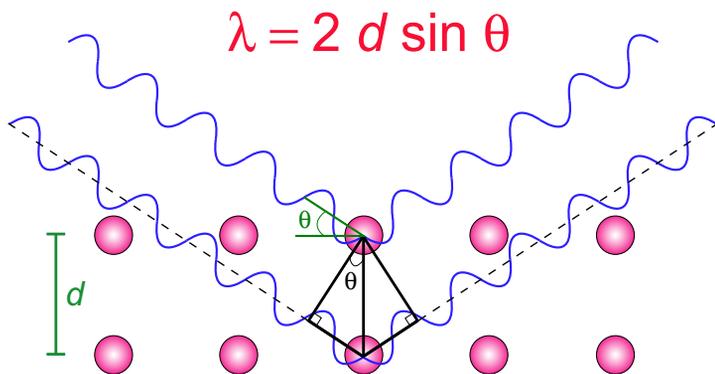
Bragg's conception of the reflection of X-ray waves from successive parallel planes of atoms to generate diffraction intensities when the interference of reflected waves is constructive.

In October, W.L. had returned to Cambridge and was strolling along the Backs of the River Cam when the epiphanic moment arrived. If X-rays were wave-like, and if the sheets of atoms in sphalerite behaved as X-ray mirrors, then the constructive interference of reflected X-rays from successive sheets would generate X-ray intensities as observed by Friedrich and Knipping (FIG. 2). And so it was that in a 1912 paper in the *Proceedings of the Cambridge Philosophical Society*, Bragg's Law was born:

$$\lambda = 2\,d\,\cos\theta.$$

In its first statement, θ was selected as the angle between the incident X-ray beam and the normal to the atomic planes—an allusion to Snell's Law of refraction in an implicit acknowledgment of the analogy between X-rays and the wave behavior of visible light. You will find no *hkl*'s in his descriptions of *d*-spacings in this first paper. Bragg's knowledge of crystallography was so thin that Miller plane notation was unknown to him.

Within a year, however, Bragg had mastered the basics of crystallography, and he recast the equation with reference to the Miller indices in the form that now is so familiar:
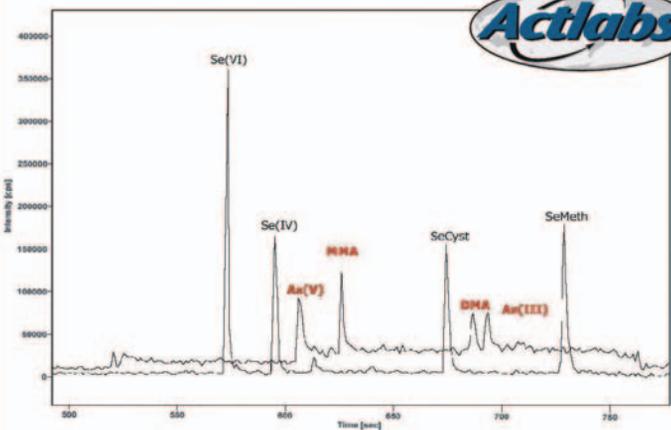
$$n\,\lambda = 2\,d\,\sin\theta.$$

As a final empirical proof, Bragg folded a book of muscovite into a circle and demonstrated that the X-rays could be focused to a point at the center. The muscovite used in these early experiments can be viewed today in a modest museum exhibit in the Cavendish Laboratory at Cambridge.

Unlike Laue, whose contributions to X-ray diffraction waned in succeeding years, Bragg and his students proceeded to unlock the crystalline universe. The structures of alkali halides and diamond followed in short order, destroying preconceptions that molecular entities must be maintained in the solid state. Bragg was the first to perceive and quantify the idea of distinct atomic radii, which bore such fruit for V.M. Goldschmidt and Linus Pauling. Bragg synthesized diffracted X-ray waveforms by Fourier methods to obtain what could rightly be called the first pictures of atoms (for diopside in 1929). And to the delight of geoscientists, he unraveled the structures of many of the major classes of silicate minerals, for which he received the Roebling Medal in 1948.

It is no surprise, then, that the story of Bragg's discovery is poorly encapsulated by the simple line drawings and terse descriptions found in most textbooks. Bragg transformed our perceptions of the invisible world as completely as did his more celebrated contemporaries. "Look at your face in the mirror—I am there inside," sings the Phantom. The reflections of W.L. Bragg can be seen in every mineral model that we use to open the atomic world to our students.